eq(iv) $\Rightarrow \gamma = x+y-z-\alpha$

$$= x+y-z-(3y-2z)$$
$$= x+y-z-3y+2z$$
$$= x-2y+z.$$

$\therefore \alpha = 3y-2z$

$\beta = z-y$

$\gamma = x-2y+z$

$\therefore$ Every elements of $\mathbb{R}^3$ can be represented as linear combination of $\{(1,1,1),(1,2,3),(1,0,0)\}$

$\therefore [B] = \mathbb{R}^3$.

$\therefore B$ is a basis for $\mathbb{R}^3$.

Q:- $V = \mathbb{R}^3$

$W_1 = \{(x,y,z) \in \mathbb{R}^3,\ x+y=0\}$

$W_2 = \{(x,y,z) \in \mathbb{R}^3,\ 2x+z=0\}$

check that W_1, W_2 and $W_1 \cap W_2$ are subspaces or not.

A:- Let $u_1, u_2 \in W_1$ such that

$u_1 = (x_1, y_1, z_1)$, $x_1+y_1=0$

$u_2 = (x_2, y_2, z_2)$, $x_2+y_2=0$

Now, $u_1+u_2 = (x_1+x_2,\ y_1+y_2,\ z_1+z_2)$, $x_1+x_2+y_1+y_2=0$.

$\cancel{x_1+x_2=0},\ \cancel{y_1+y_2=0}$

$\alpha u_1 = (\alpha x_1, \alpha y_1, \alpha z_1)$, $\alpha x_1+\alpha y_1=0$

$\therefore u_1+u_2 \in W_1\ \forall u_1, u_2 \in W_1$

$\alpha u_1 \in W_1\ \forall \alpha \in F,\ u_1 \in W_1$

$\therefore W_1$ is a subspace of V.