$\therefore$ Now $u+v = (\alpha_1 u_1 + \dots + \alpha_n u_n) + (\beta_1 u_1 + \dots + \beta_n u_n)$

$$= (\alpha_1+\beta_1)u_1 + \dots + (\alpha_n+\beta_n)u_n$$

$$= \gamma_1 u_1 + \dots + \gamma_n u_n \in [S]$$

$$\alpha u = \alpha(\alpha_1 u_1 + \dots + \alpha_n u_n)$$

$$= \alpha\alpha_1 u_1 + \dots + \alpha\alpha_n u_n$$

$$= c_1 u_1 + \dots + c_n u_n \in [S].$$

$\therefore$ $[S]$ is a subspace of v.s. $V$. □

**Theorem:-**

If $S$ is a non-empty subset of a vectorspace $V$, then $[S]$ is the smallest subspace of $V$ containing $S$.

**Proof:-**

(i) $[S]$ is subspace of $V$.

(ii) $S \subseteq [S]$

(iii) $[S] \subseteq W$

Let $\{u_1, u_2, \dots, u_n\} = S$

$S \subseteq W$ (Assume)

$\Rightarrow \{u_1, u_2, \dots, u_n\} \subseteq W$

$\Rightarrow \alpha_1 u_1 + \alpha_2 u_2 + \dots + \alpha_n u_n \in W$

$\Rightarrow [S] \subseteq W.$